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LITTLE JOE II PROGRAM

MONTE CARLO STUDY OF MOTOR PERFORMANCE

Prepared by

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MONTE CARLO STUDY FOR LITTLE JOE II MOTORS

Enclosures: (1) Values Used for the Little Joe Monte Carlo Program

(2) Chamber Pressure vs Time at 70° - 100 Runs

(3) Sea Level Thrust vs Time at 70° - 100 Runs

(4) Altitude Thrust vs Time at 70° - 100 Runs

(5) Nominal Thrust and Pressure at 50, 70, and 90°F at Sea Level

(6) Nominal Thrust and Pressure at 50, 70, and 90°F at Altitude

(7) Three Sigma Bands of Thrust and Pressure at 70°F and Sea Level

(8) Three Sigma Bands of Thrust and Pressure at 70°F and Altitude

(9) Thrust and Pressure at 70°F from Sea Level to 100,000 Feet

SUMMARY

The Monte Carlo study consisted of a series of ballistic evaluations to determine the effects of propellant variability upon ballistic performance of Algol ID motors for the Little Joe II Program. To determine the performance variations, values of selected propellant ballistic properties were sampled randomly from a normally distributed population of known mean and standard deviation. One hundred ballistic evaluations were made using the randomly sampled properties and the resulting performance curves are plotted in Enclosures 2 through 9.

Technical Discussion

The mathematical model used for the Monte Carlo study consists of Aerojet Computer Program 1103 together with the input description of the Algol ID motor which provided an adequate simulation of the LJ-4 motor. This model was selected because it provided the best possible prediction of the ballistic behavior of the Little Joe II motors.

The following characteristics were subjected to random variation for the Monte Carlo Program. The standard deviations were computed from the 100 values of these characteristics used for the program.

Parameter	Mean Value	Range	Standard Deviation
ρ - Density	0.06034	0.06014-0.06054	0.0000670
C_w - Flow Coefficient	0.006600	0.006255-0.006945	0.000117
C - Burning Rate Constant	0.08503	0.08033-0.09000	0.001546
A_t - Throat Area	175.33	175.00-175.66	0.0320

These characteristics were selected for variation in the Monte Carlo program because they produce the only significant variations in motor-to-motor performance.

Of these variables, burning rate changes produce the greatest performance variations.

The range of values for the burning rate constant (0.08033 to 0.09000) was taken from the propellant specification, WS-1013, which specifies burning rate limits of 0.223 to 0.265 inches per second. The relationship $r = CP^n$ is used to convert the burning rate, r, to the burning rate constant, C, which is used in the computer program. The actual limits for the burning rate, r, among the seven Little Joe II motors produced, were 0.245 to 0.256 inches per second. These actual limits are equivalent to a burning rate constant range of 0.0833 to 0.0870 which is a narrower range of burning rates than allowed by the propellant specification and used in this Monte Carlo program. This motor burning variation is explained by the number of batches of propellant in a batch mixed rocket motor. The burning rates of the ten batches of propellant may vary within the limits allowed by the propellant specification. However, the average burning rate of the ten batches is used for the prediction of motor performance. In the case of continuously mixed motors, the burning rate of the propellant throughout the motor is more uniform.

The range of values for the density (0.06014 - 0.06054 pounds per cubic inch) was taken from the propellant specification. The actual limits for the density of propellant among the seven Little Joe II motors produced were 0.06030 to 0.06037 pounds per cubic inch. Again, the actual motors produced cover a much narrower band of values than that permitted by the specification. As with the burning rate, the motor propellant density used for performance predictions is an average of the densities of the ten individual batches of propellant in the motor.

The range of values for the nozzle throat area was taken from actual measurements of nozzle throats that were used on Algol ID motors that were fired. The range of values for the mass flow coefficient was taken from data on fired Algol ID motors.

The random normally distributed samples for the program were generated by a computer program which combines the method of congruences and the central limit theorem. The 100 values generated for the Little Joe II Monte Carlo Program are listed in Enclosure 1.

The standard deviations are based on batch variations, and therefore they describe the variability which would be expected if each motor were cast with one batch. However, since each Algol is made with ten batches of propellant, the standard deviation for each parameter (σm) should be

$$\sigma_M = \frac{\sigma_B}{\sqrt{10}}$$

where σ_B is the batch standard deviation listed in the chart above. Because of a likelihood that adjacent batches will be more nearly alike than batches cast at widely spaced intervals, there is probably a bias tending to increase the value of σm. Therefore, the true value of standard deviation falls somewhere between the batch standard deviation and σm calculated by the above equation.

The performance curves for the 100 simulated motor firings at 70°F are plotted on Enclosures 2 through 4. The nominal curves for 50, 70, and 90°F are shown on Enclosures 5 and 6. The three sigma bands of thrust and pressure at sea level and altitude conditions are shown in Enclosures 7 and 8. The nominal thrust at 10,000 foot altitude increments from 10,000 through 100,000 is shown in Enclosure 9.

From the preceding discussion it can be concluded that the dispersion observed in the Monte Carlo runs is greater than can be expected in actual firings and represents an extreme limit. Below is a list of significant ballistic factors, their mean values, and their standard deviations:

Parameter	Mean Value	Standard Deviation
Initial Pressure, psi	474.10	10.36
Maximum Pressure, psi	484.90	10.75
Maximum Thrust, lbf	123,030	3175.0
Duration, sec	34.41	0.676

The performance curves plotted in Enclosure 2 through 4 show one curve which is considerably higher than the others. This particular run is case 31, with an unusually high burning rate constant of 0.090474. The four factors cited above, have the following values for this curve: 512.803 psi, 525.745 psi, 134,643 lbf, and 32 seconds. These values are not within three standard deviations of the mean value; however, this is the only case which is not within the three sigma band.

According to statistical theory, 99.7% of cases should be between plus and minus three sigma. It is unusual but statistically possible that one case out of one hundred would fall outside this range. The probability that out of one hundred firings, at least one will produce a curve this far from the mean is 0.168.

A study was made of the correlation between the four characteristics used as input and the four factors selected to evaluate the results. The following table lists the four output factors down the page, the four input characteristics across the top, and the square of the correlation coefficient in the spaces. This value represents the fraction of the dispersion of each resulting parameter which is attributable to each input parameter. For example, in the first line, 99.37% of the dispersion in initial pressure is caused by variations in the burning rate constant C. The total does not equal exactly one, because of the limited sample.

	C	C_w	ρ	A_t	Total
P_i	0.9937	0.0068	0.0086	0.0001	1.0092
P_{max}	0.9900	0.0066	0.0077	0.0001	1.0044
F_{max}	0.9908	0.0001	0.0084	0	0.9993
t_b	0.9482	0.0078	0.0001	0	0.9561

It is evident that the burning rate constant C is the only parameter which significantly affects the pressure, thrust, and duration. The squares of the correlation coefficients for duration are less than one because the definition of burn time is inexact. A more careful definition of burn time will give a more satisfactory correlation.

Conclusions

1. The standard deviations of the thrust, pressure and duration as calculated from the Monte Carlo runs is undoubtedly higher than would be expected from a series of 100 actual motor firings, because of the ten-batch factor discussed above. Therefore, the standard deviation of about 2% is perhaps high.

2. The Monte Carlo technique represents the best method known at this time for assessing the effects of variability in the components of a solid rocket motor. The performance evaluations resulting from this study give a prediction of the behavior of 100 randomly sampled Algol ID motors produced within the specification limits noted. However, the seven Little Joe II motors produced were within a much narrower range of density and burning rate values than allowed by specification.

3. Individual motor predictions should be used for planning vehicle performance. Such predictions are based on nominal performance, modified by the average burning rate of the propellant in the individual motor. The validity of the prediction system has been demonstrated by the test firings of Motor LJ-7 and the first Little Joe II vehicle at White Sands.

DENSITY	MASS FLOW COEFFICIENT	THROAT AREA	BURNING RATE CONSTANT
.060356	.006399	175.328707	.085771
.060327	.006778	175,329025	.086550
.060432	.006628	175.330120	.086550
.060337	.006601	175.328747	.085358
.060428	.006602	175.328547	.085814
.060291	.006624	175.329533	.084874
.060329	.006478	175.332729	.086176
.060199	.006634	175.328318	.086205
.060302	.006412	175.330387	.083014
.060461	.006501	175.327848	.083207
.060387	.006676	175.328468	.084089
.060382	.006671	175.330788	.084030
.060307	.006442	175.329216	.084591
.060450	.006532	175.329241	.083345
.060311	.006633	175.329203	.085858
.060381	.006615	175.329729	.086774
.060458	.006553	175.330301	.083556
.060313	.006700	175.330942	.085390
.060315	.006635	175.329538	.084021
.060416	.006475	175.331779	.084238
.060382	.006518	175.329285	.081067
.060379	.006604	175.329765	.085420
.060327	.006410	175.331118	.086863
.060261	.006609	175.329632	.084848
.060412	.006340	175.328838	.082332
.060299	.006504	175.330025	.085311
.060297	.006522	175.329548	.084049
.060363	.006603	175.330450	.085954
.060329	.006570	175.329729	.083883
.060442	.006486	175.330936	.083724
.060378	.006533	175.330013	.090474
.060342	.006615	175.330322	.086149
.060272	.006661	175.329527	.085738
.060297	.006774	175.332142	.085061
.060336	.006644	175.328350	. 83442
.060379	.006316	175.330259	.084765
.060286	.006457	175.330967	.086278
.060243	.006614	175.329714	.084587
.060375	.006565	175.329675	.084745
.060296	.006592	175.330452	.086885
.060455	.006614	175.330559	.082906
.060385	.006549	175.330912	.082623
.060312	.006556	175.328306	.083432
.060390	.006655	175.331236	.086699
.060304	.006732	175.330591	.085996
.060437	.006824	175.329765	.083670
.060345	.006598	175.329277	.085149
.060215	.006438	175.328472	.084686
.060385	.006866	175.329714	.085313
.060239	.006714	175.329575	.086687

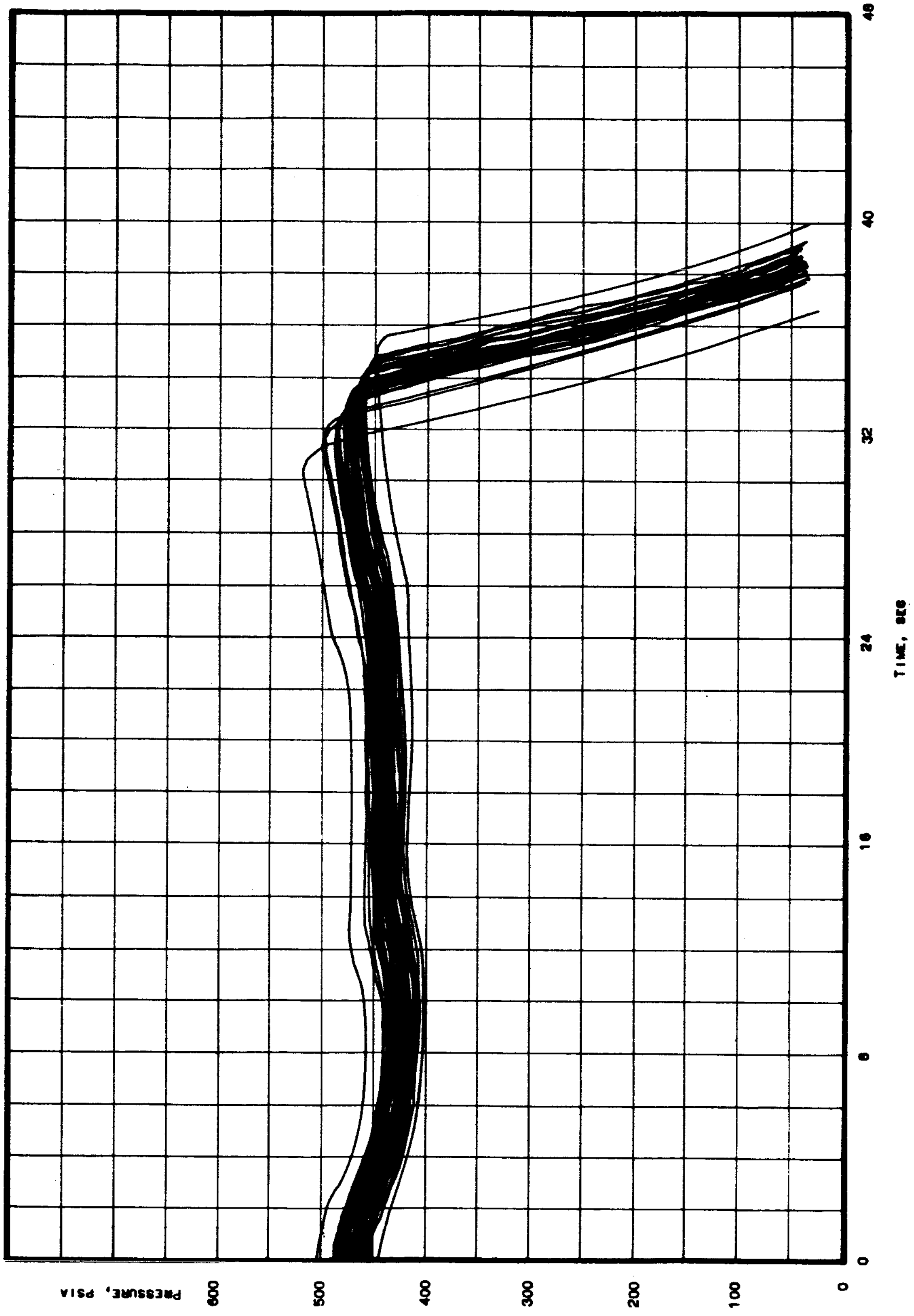
Values for Characteristics Used in Computer Program

DENSITY*	MASS FLOW COEFFICIENT	THROAT AREA	BURNING RATE CONSTANT
.060396	.006681	175.329145	.085883
.060407	.006538	174.331003	.084336
.060259	.006744	175.330076	.085374
.060187	.006523	175.331028	.083196
.060395	.006693	175.330542	.085270
.060371	.006544	175.327299	.086287
.060237	.006762	175.329830	.082026
.060236	.006644	175.331028	.083929
.060323	.006615	175.331562	.084703
.060402	.006699	175.327579	.084924
.060310	.006662	175.329529	.084977
.060294	.006402	175.330971	.084307
.060308	.006439	175.328012	.085042
.060362	.006748	175.330723	.084111
.060403	.006678	175.329556	.085416
.060395	.006522	175.329481	.084780
.060308	.006523	175.329870	.083510
.060282	.006487	175.330181	.084127
.060315	.006404	175.328957	.082309
.060296	.006729	175.330181	.085406
.060330	.006515	175.329542	.088753
.060225	.006413	175.329260	.086624
.060343	.006751	175.332266	.082676
.060358	.006577	175.332376	.086483
.060390	.006685	175.331135	.087744
.060345	.006685	175.328791	.085144
.060418	.006615	175.328390	.083762
.060221	.006638	175.329311	.084013
.060344	.006836	175.330984	.085218
.060394	.006526	175.327396	.083887
.060372	.006625	175.327736	.085437
.060397	.006614	175.330183	.083966
.060385	.006553	175.330116	.083407
.060383	.006691	175.330036	.082314
.060255	.006547	175.329887	.083326
.060271	.006836	175.329800	.087901
.060470	.006448	175.330328	.082262
.060394	.006476	175.329187	.087531
.060337	.006608	175.330364	.084285
.060313	.006546	175.330582	.084204
.060363	.006456	175.329168	.086330
.060331	.006660	175.330273	.085850
.060399	.006734	175.329914	.083239
.060255	.006761	175.332056	.084355
.060469	.006801	175.330706	.084221
.060340	.006633	175.328684	.085862
.060352	.006726	175.330296	.086258
.060370	.006661	175.330511	.085890
.060455	.006798	175.330774	.082844
.060219	.006683	175.329521	.086329

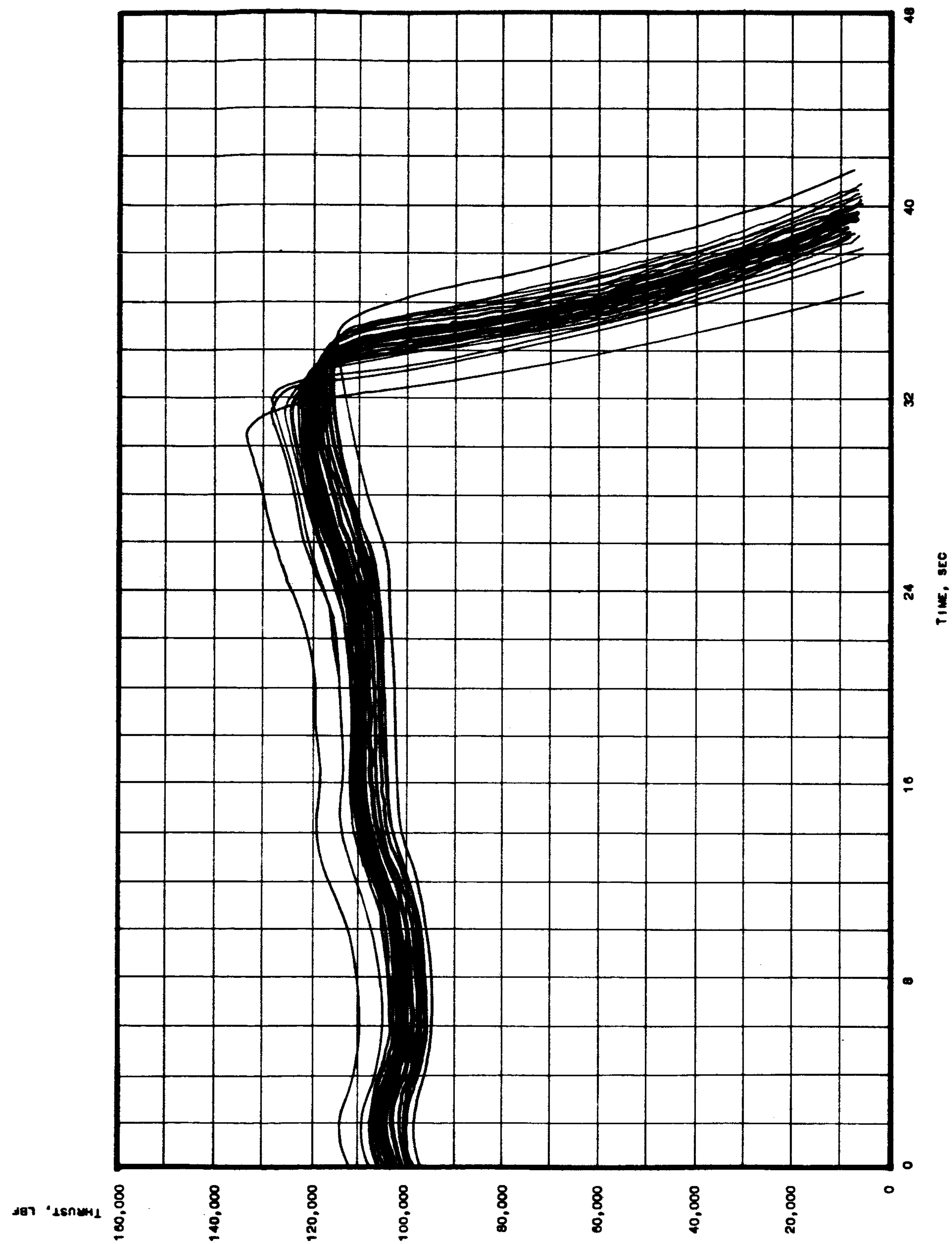
Values for Characteristics Used in Computer Program

Monte Carlo Study

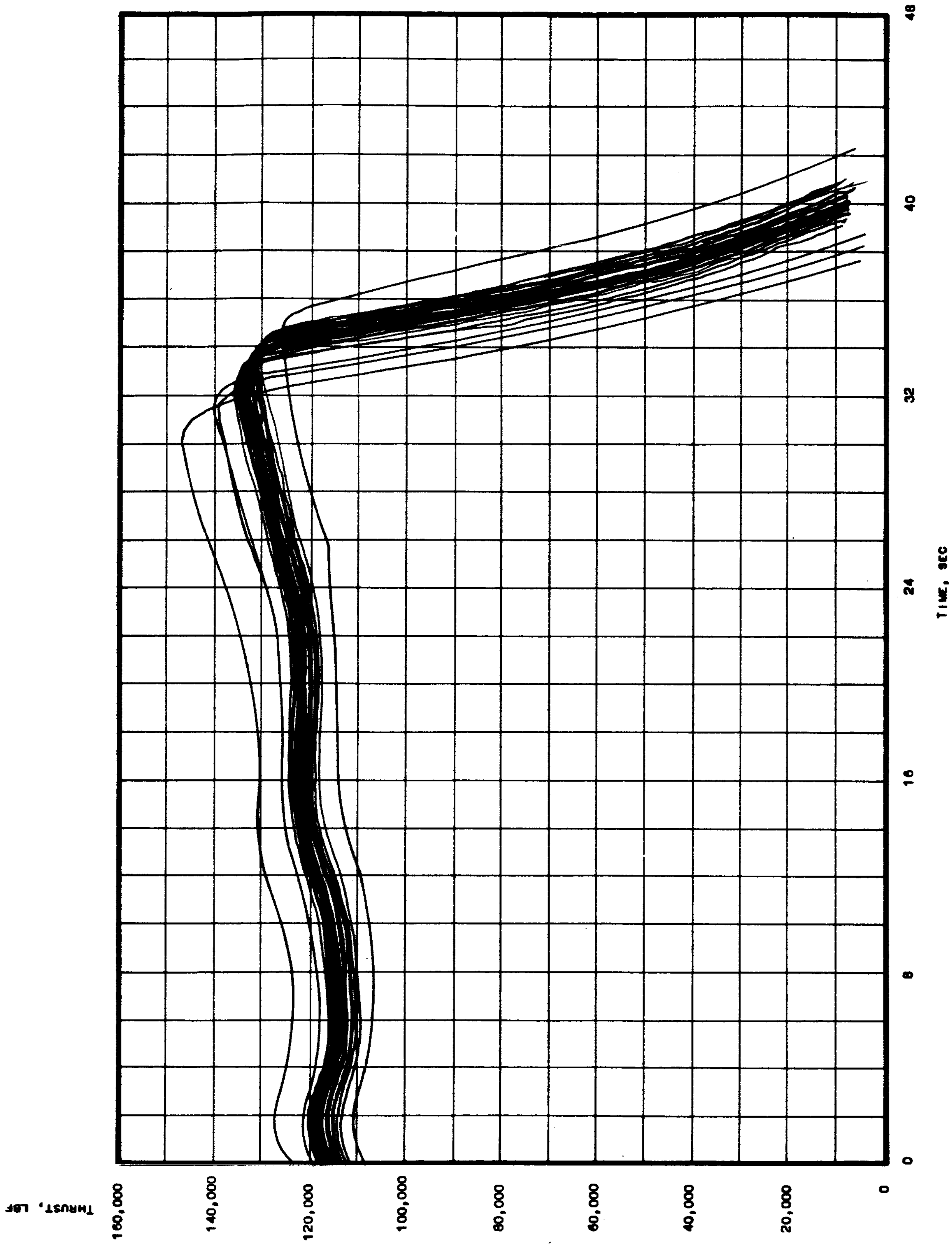
Enclosure 2



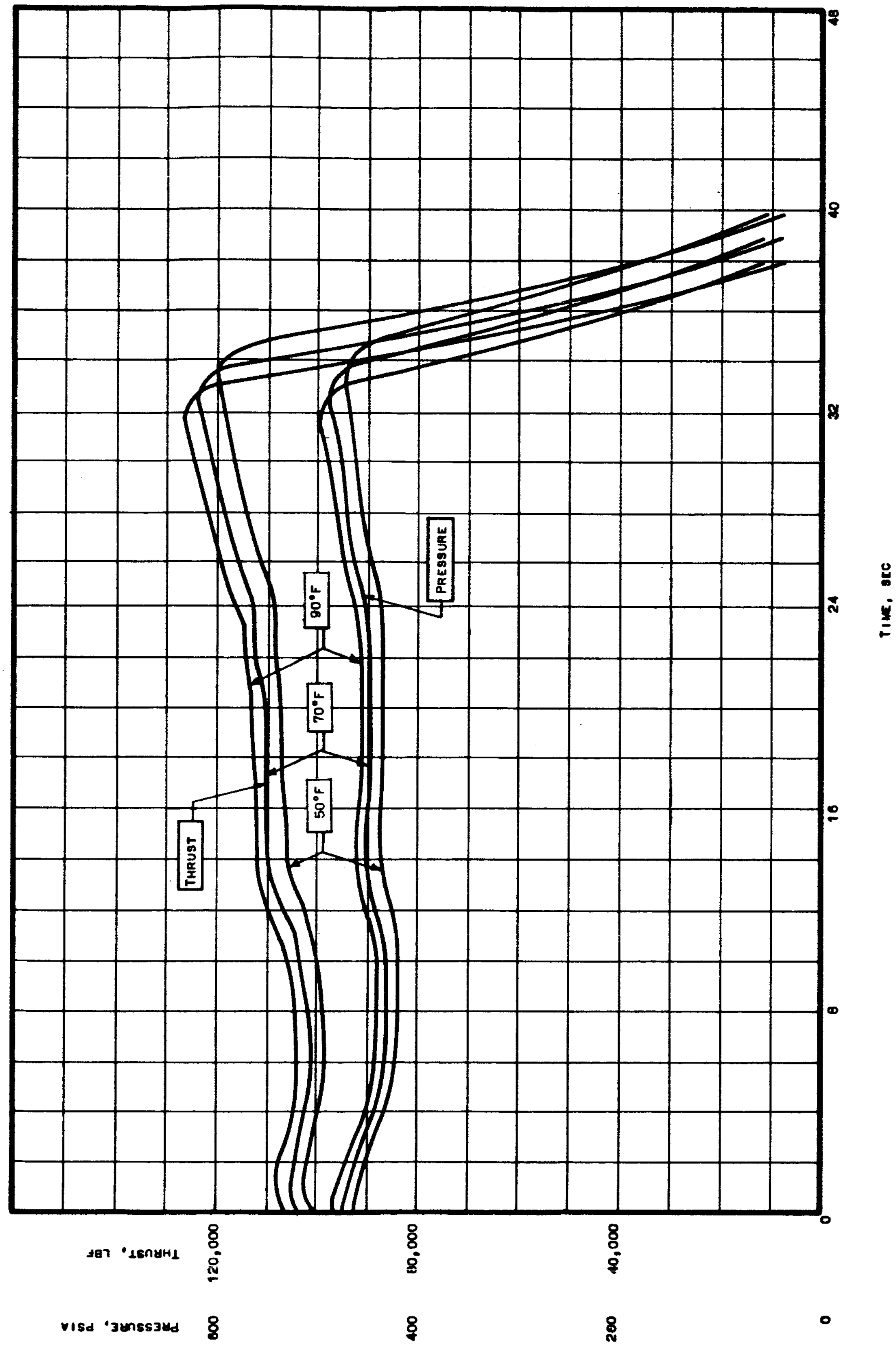
Chamber Pressure vs Time at 70°F, 100 Runs



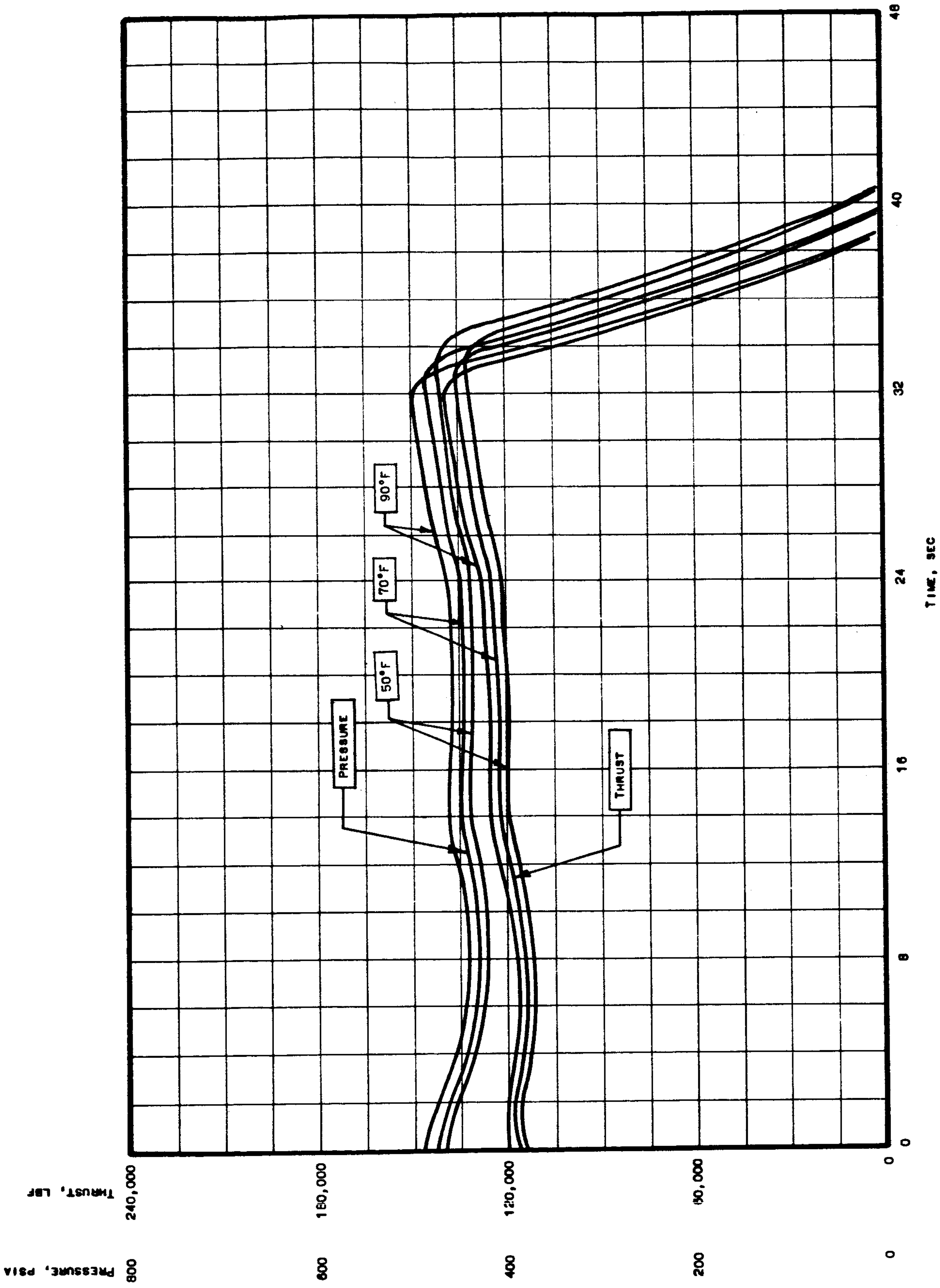
Sea-Level Thrust vs Time at 70°F, 100 Runs



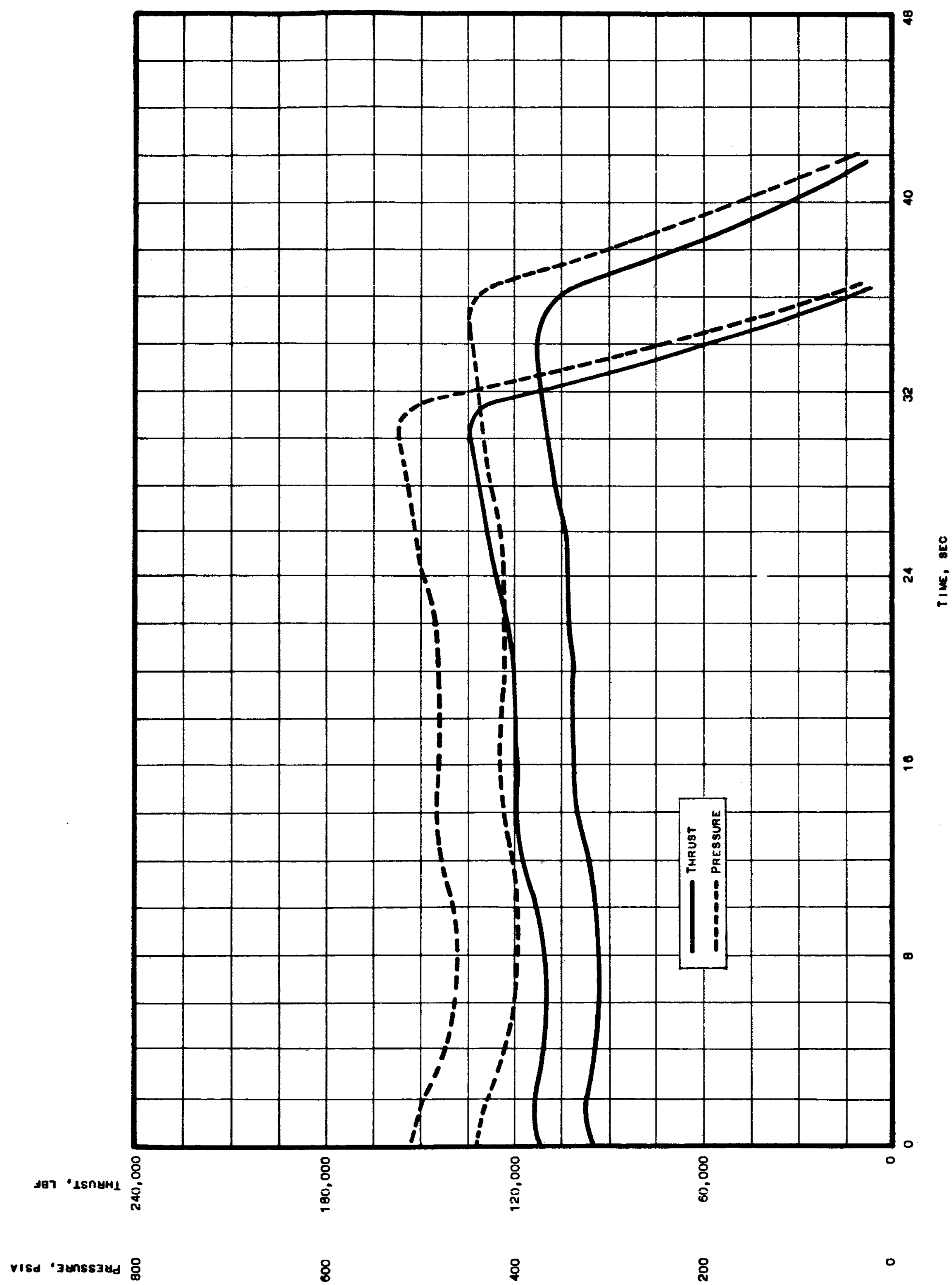
Altitude Thrust vs Time at 70°F, 100 Runs



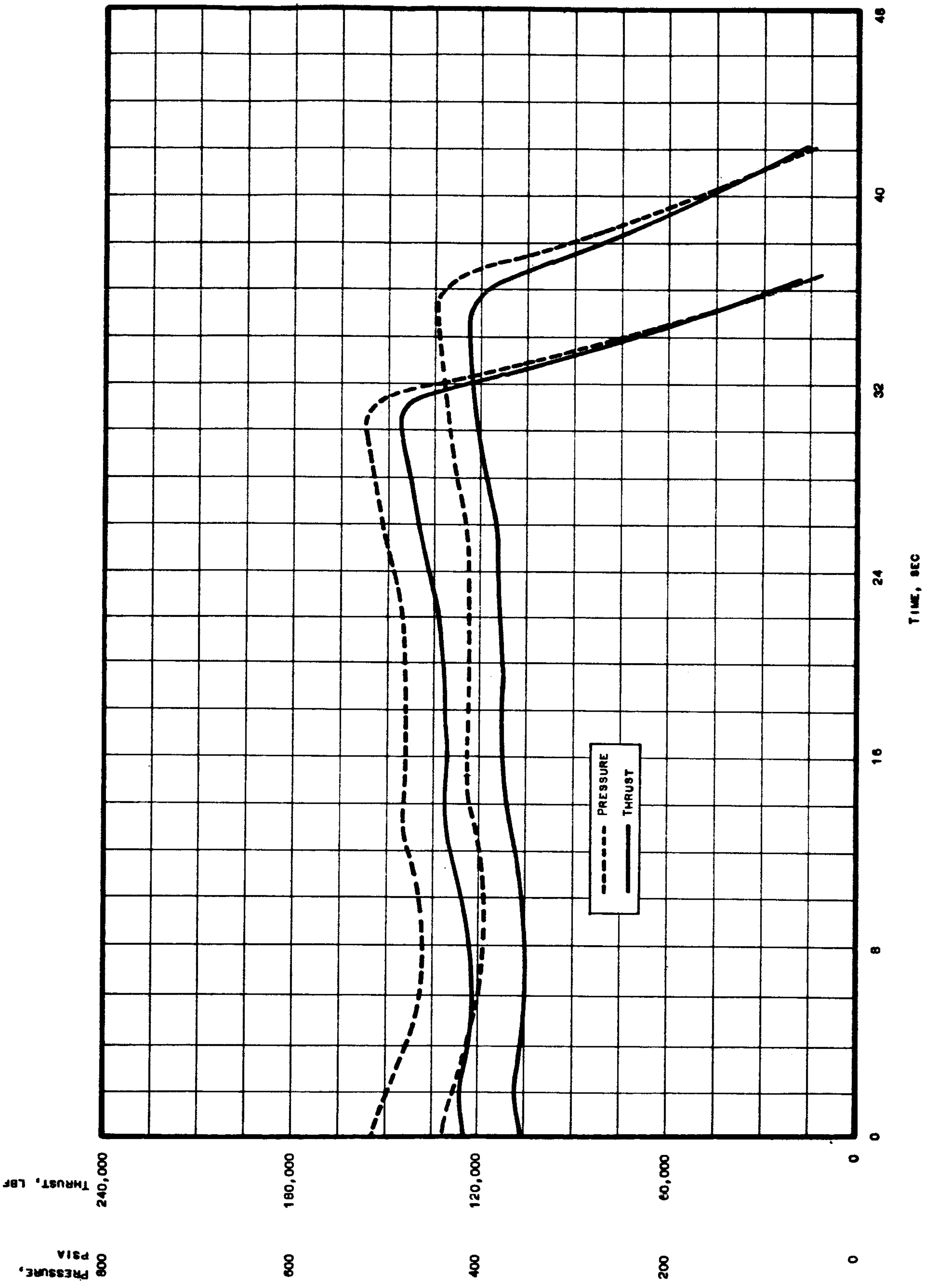
Nominal Thrust and Pressure at 50, 70, and 90°F at Sea Level



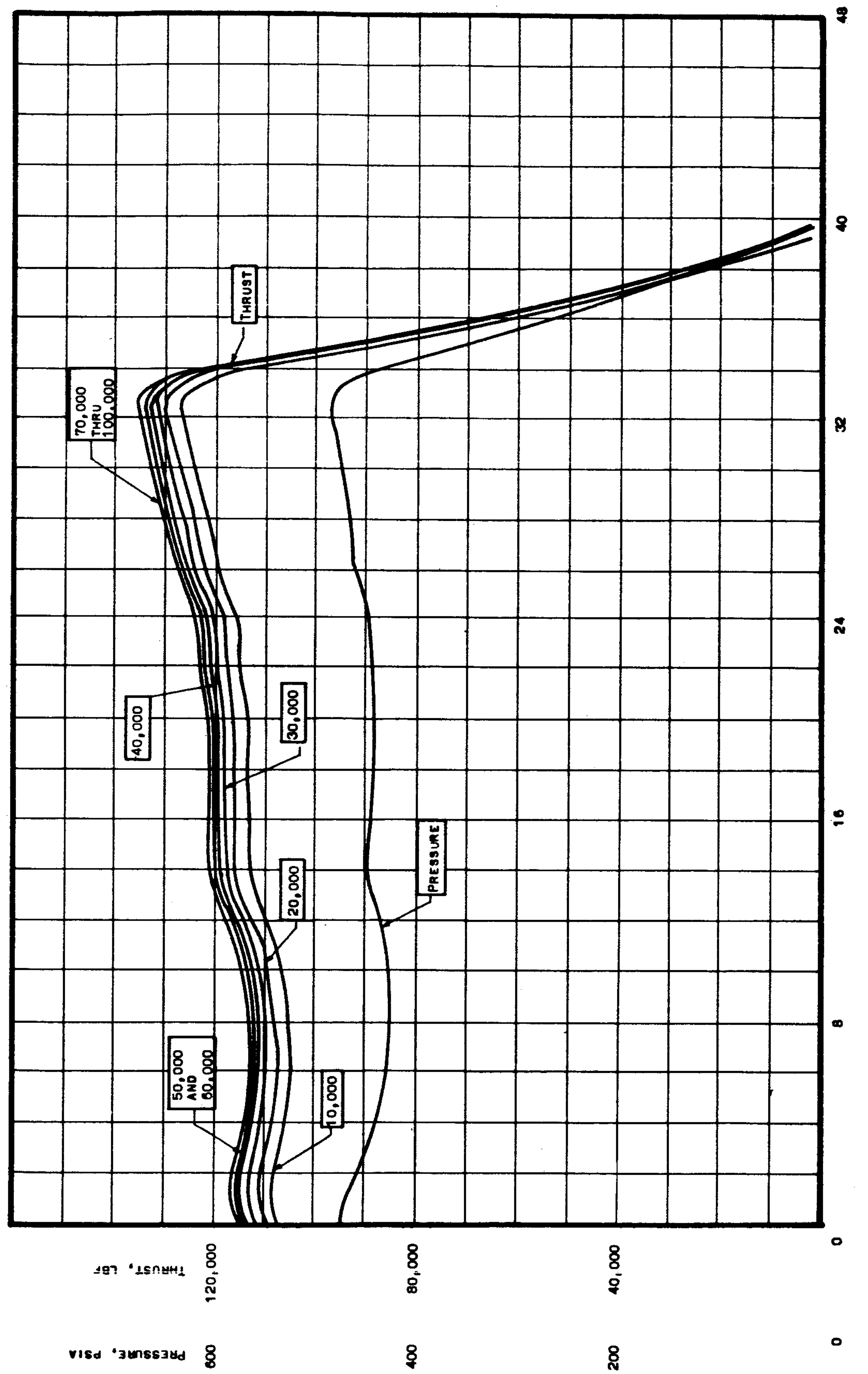
Nominal Thrust and Pressure at 50, 70, and 90°F at Altitude



Three-Sigma Limits of Thrust and Pressure at 70°F and Sea Level



Three-Sigma Limits of Thrust and Pressure at 70°F and Altitude



Thrust and Pressure at 70°F From Sea Level to 100,000 ft